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Abstract	The problem of cognitive dissonance in the aspect of automated solution of planimetric problems is investigated. Algorithms based on the ontology of the subject area and aimed to the elimination of cognitive dissonance are developed. Computer implementation of the developed algorithms with linguistic and visual aid is investigated. The generality of the approach of eliminating cognitive dissonance in solving problems from other domains in the presence of an appropriate ontology is highlighted. The significance of fundamental and applied components of the investigated problem and the importance of the developed approach for the formation of students’ critical thinking are emphasized.

Keywords (separated by '-')	Cognitive dissonance - Planimetric problem solver - Domain ontology - Linguistic and visual solution aid

Cognitive Dissonance in Solving Planimetric Problems

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Abstract. The problem of cognitive dissonance in the aspect of automated solution of planimetric problems is investigated. Algorithms based on the ontology of the subject area and aimed to the elimination of cognitive dissonance are developed. Computer implementation of the developed algorithms with linguistic and visual aid is investigated. The generality of the approach of eliminating cognitive dissonance in solving problems from other domains in the presence of an appropriate ontology is highlighted. The significance of fundamental and applied components of the investigated problem and the importance of the developed approach for the formation of students' critical thinking are emphasized.

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1 Introduction

Cognitive dissonance is defined in Wikipedia as a state of mental discomfort of an individual caused by the clash of conflicting perceptions in his consciousness. Cognitive dissonance (CD) is a complex and multifaceted phenomenon, that is investigated by very different scientific approaches including psychology, physiology, education, sociology, etc. For example, psychology builds cognitive network models that predict changes in people's beliefs about a controversial scientific topic [2]. Russian scientists have proposed a new approach to the study of cognitive dissonance at the level of physiology using magnetic stimulation online. Learning in the conditions of online communications takes into account the specificity of the educational environment, the inclusion of cognitive dissonance in learning models promotes awareness of psychological conflicts, critical perception of new facts and information [10].

The difficulty of modeling human physiological and psychological states by AI methods (in the aspect of analysis) is noted in [9]. It is hypothesized that introducing cognitive dissonance into artificial intelligence (AI) models will be necessary to create "strong" AI. The psychological aspects of cognitive dissonance associated with the accelerating trend towards the use of large language models (LLMs) and the dangers of using AI in social domains are discussed in [5]. From a philosophical perspective, it is discussed that CD is what makes the brain generate creative solutions. In linguistics, extralinguistic and linguistic criteria for identifying cognitive dissonance are studied.

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The article discusses cognitive dissonance in the aspect of solving planimetric problems and ways to overcome it in a computer solver of such problems [12, 13]. The relevance of the study is determined by the fact that although automatic solvers have achieved impressive success in the actual solution of problems [18], but in the aspect of education the problem of CD is not covered by solvers in a number of significant aspects. The automation of solving is explored in the framework of the heuristic concept of the famous scientist and educator Pólya [16], but the question of the relationship between axiomatics and experimental data, as well as the problem of non-Euclidean geometry is touched upon. It is known what kind of cognitive dissonance Lobachevsky's publication on this problem caused in the scientific community. In the framework of computer realization, the issues of overcoming CD are discussed.

In human solution of a geometric problem, CD can arise as a mismatch of an external factor with an internal knowledge system. At the linguistic level, the CD can arise as a discrepancy between the natural-linguistic description of the task conditions and the internal understanding of the object. For example, "... The triangle formed by these tangents and another internal tangent...". The student should realize that a triangle can be formed by three lines, "one more" means that one inner tangent has already been used and, finally, a circle has two inner and two outer tangents.

2 Methods and Tools

2.1 Methodology

The paper investigates the problem of computer modeling of CD in the process of interaction with a system for solving planimetric problems at different stages: linguistic translation, automated solution, and visualization. Therefore, known methods based on modeling CD using a weighted oriented graph have not been directly used. However, methods for working with graphs representing fragments of specified ontology fragments were of course used in overcoming CD. Such fragments included concepts defining the decision situation and oriented edges corresponding, for example, to logical sequence links between substructures.

The methodology of the system functioning is based on the cognitive schemes presented in the ontology, which integrate knowledge about natural language, subject domain and visualization tools. It is the schemes that ensure the integrity of processing by modeling the cognitive process of understanding the text of a natural language task, its representation in the preliminary drawing and plausible reasoning during the solution. The theoretical basis of geometry is well known; important completeness and solvability theorems have been proved. The difficulties of practical use of theoretical results and the necessity of relations of content-semantic nature for successful deduction are also known. The ontology fixes the theoretical basis at a level of rigor sufficient for the purposes of the study (somewhat higher than a school textbook).

The problem of CD is very complicated and the more complicated is the question of its computer modeling by AI methods. The choice of automating the solution of geometric problems is important due to the fact that the system under study considers overcoming CD at a number of stages of functioning: linguistic processing, solution, visualization. The system is based on Poya's principles, which have not lost their importance nowadays. An analysis of these principles convinces us that Poya emphasized the cognitive aspects of the problem. The relative simplicity of linguistic aspects, the naturalness of semantic representation, and the interactivity of visualization allow us to focus on the key points of overcoming CD without diluting the essence of the problem. Computer realization of heuristics has been performed many times, but the formation of super-high-level heuristics (common sense) and their complex integration with both deduction and plausible reasoning remains one of the central problems of artificial intelligence. Our research is scientifically oriented towards this very problem; an approach based only on neural networks (and LLMs) seems insufficient to us.

From philosophical positions, let us note that the question of the Earth's shape can be interpreted as CD overcoming in relation to geometrical shape (flat or round). An even more vivid example is Lobachevsky's geometry, which made it possible to complete the CD, which had attracted the attention of scientists for 2000 years. In the process of informal communication with students, using an analogy with the question of the Earth's sphericity, it is possible to overcome the CD regarding the axiom of parallelism. The students agree that the fairness of this axiom requires experimental confirmation by measuring the sum of angles of a triangle. In this case, inaccuracy of measurement can lead to erroneous conclusions (as when measuring small areas of the Earth). Of particular importance in the applied aspect is the possibility to familiarize students with the elements of artificial intelligence in specific situations directly related to the educational process ((“understanding” by the computer of the task text, automation of the solution and its visualization). We assume that the openness of the system texts allows making modifications at the level of laboratory works in computer classes. The initial version used a database with SQL language and cumbersome ontology. The current implementation specifically uses simple tools and a simplified processing style (EXCEL and VBA macros, JavaScript and the powerful JSXGRAPH visualization library) to demonstrate as clearly as possible how AI elements are used to solve geometric problems formulated in natural language (NL). EXCEL tables allow to show the process of transition from the NL description of the problem to the internal representation by simple paraphrasing operations, the reference to a theorem - as a call to a subroutine with parameters. Complex issues of NL processing (ellipsis, anaphoric references, etc.) and ontology organization can be ideologically quite accessible to students, especially when they are related to the learning process. Specifically, when analyzing the elliptical (incomplete) sentence, “The median of the first triangle is 16 cm and the median of the second triangle is 25”, students not only explain how this sentence is understood, but also realize the need for a general schema to understand similar sentences (heights, angles, radii, etc.). It is the ideational essence that is important, not the technical side (representing the schema in an ontology, signifying schema parameters, etc.).

Plausible reasoning in simple variants contains references like “less/more familiar shapes or angles”. In the complex case of the following Olympiad problem, the plausible reasoning for advanced students was “Inversion is appropriate to use because it can map circles into straight lines. It may be easier to prove the relation between straight lines than to prove the relation between circles. There are 4 circles mentioned in the problem text”. Here again the idea side is important (technical, the center of inversion and which inversion property to use). Empirical guesses based on the drawing and interactive visualization capabilities do not require much commentary. To justify the necessity of proving an empirical statement, students were given an example from life: it is not enough to "EXPLAIN" a criminal, it is necessary to "PROVE" his guilt. The analogy to calculated angles on blueprints is obvious.

The used methods of automatic proof (provers) are substantially supplemented by algorithmic and software development of Pólya's concept of plausible reasoning, including natural language-oriented justification of solution steps and interactive visualization. Pólya's concept is developed in 3 basic works, but it is addressed to humans and its computer implementation requires serious effort. Pólya emphasizes the cognitive discomfort that arises in the learner when a correct, efficient, and short solution is demonstrated – "...the solution appears out of the blue, pops up from nowhere." [16]. Another type of cognitive discomfort occurs in schoolchildren when proving a theorem that is acceptable only for students of the faculty of mathematics. According to Pólya "advanced" high school students can openly declare that it was obvious to them what was being proved and it did not become more obvious after the "proof". Interaction with the system in which the CD is considered provides the student with ample opportunities to overcome the CD: to analyze the conditions of the problem in a dialog, to guess a part of the solution, to construct a correct drawing, etc. Such possibilities, of course, were not considered by Pólya.

To conclude the section, let us briefly formulate the idea side of the approach. The described cognitive approach to automated solution of planimetric problems involves input of the problem text in EJ, linguistic processing, automatic solution and visualization of the result within a computer system.

The cycle of the system operation: "**NL description of the problem**" => "**UNDERSTANDING TEXT**" => "**SOLUTION**" => "**DRAWING**".

At each stage of the system operation different methods of **overcoming CD** are used.

At the linguistic stage - **paraphrasing method**, which allows to reduce equivalent NL descriptions to a canonical form.

At the solution stage, the **Poya heuristic method**.

At the visualization stage, the **drawing modification method**, which focuses on the transformation of the drawing (while preserving the problem conditions), on the labeling of key elements, and on empirical guesses that significantly facilitate the solution.

Specifically, at the paraphrasing stage, the CD allows us to move to more comprehensible formulations ("segment from vertex to the middle of the side opposite" = >"median").

At the solution stage - "right triangle" => "Pythagoras' theorem".

At the visualization stage - "it is reasonable to connect points A and B to obtain a familiar figure". Modification of the drawing is aimed at modeling human actions corresponding to the NL description: "change the sides of the triangle and make sure that the conclusion of the problem is preserved".

The overall functioning of the system is based on the cognitive schemas presented in the ontology, which integrate knowledge about natural language, subject domain and visualization tools. It is the schemas that provide **processing integrity** by *modeling the cognitive process of understanding* the NL text of the problem, its representation in a preliminary drawing and plausible reasoning during solution. In the current implementation, the level of abstraction of the CD is chosen to match the specifics of the subject area. For example, when analyzing the text fragment "a triangle formed by these tangents and one more internal tangent", the system is based on knowledge about tangents (two external and two internal tangents). A full-fledged cognitive schema for "one more" requires the involvement of set-theoretic considerations.

Poya's method is originally addressed to humans, its computer implementation is very complex and is far from complete within the framework of this system. Importantly, once a solution is obtained, the system generates a protocol that provides a step-by-step solution in different directions. When viewing the protocol, both a description of the system's action during the solution and the corresponding display on the drawing are given.

2.2 General Information About the System

The system, which investigates the phenomenon of CD and the possibilities of overcoming it, provides: linguistic processing of the task text, formation of a semantic representation, problem solving based on the Pólya concept, visualization of the drawing and the solution protocol. CD can occur at any stage of interaction. The overcoming of CD can be done by the system, by the student (in dialog) and by the teacher. Examples illustrating such situations are discussed further in Sects. 3 and 4.

At the linguistic stage, the task text is translated into standard descriptions of basic operations directly interpreted into a JavaScript program. When this program is executed, a drawing corresponding to the task conditions is visualized. In addition, a semantic structure of the problem is formed. The solver operates on this structure and modifies it accordingly and augments it with JavaScript code. At the end of the solver's work, the resulting JavaScript program is executed and the drawing with the solution is displayed. Viewing the solution protocol allows you to trace all the solver's actions with their justification step by step. The level of basic operations roughly corresponds to the level of operations described in [11], but is more natural language oriented.

Examples of operations: Construct the triangle MFK, Construct a circle okr_1 circumscribed about triangle MFK, Draw a straight line pr5 through point K, perpendicular to O1_K, Denote the calculated angle EO1K - °,0.20. The parameters in the last operation specify that the angle is calculated in degrees with an arc size of 0.20. The execution of these operations in a JavaScript program is supported by the JSXGraph library. A higher level is related to the use of cognitive schemas integrating knowledge of natural language, semantic representation, domain knowledge (geometry theorems) and interactive visualization. For the purposes of overcoming CD, the library of these operations has been extended and the necessary fragments for describing and invoking the operations have been added to the ontology.

3 Experiment

The experiment was conducted within the framework of a system that allows automating the solution of planimetric problems [12, 13]. From the previously tested problems, we selected the problems whose solution protocol contains not only justifications of inductive steps, but also justifications of drawing modification. In particular, this concerned compliance with the conditions of the problem in the drawing (e.g., perpendicularity of medians), elimination of drawing deficiencies (angles or segments too small, bulky, false inductive assumptions). The solution steps include choosing the style, color, and visibility of objects in the drawing depending on the selected theorem.

The most interesting are the results of the experiment on the Olympiad problem [18], which is currently evaluated as an impressive success in the automatic solution of geometric problems. Note at once that our system was not planned for solving problems of such complexity (the goals of the system: input of the problem text in natural language, orientation on the Pólya methodology and on learning tasks in the interactive visualization mode). Nevertheless, testing of the system on this task gave interesting results. Figure 1 shows two fragments to explain AlphaGeometry's working style. The left fragment is interesting by demonstrating the cognitive discomfort of an ordinary Russian schoolchild to prove the equality of angles in an isosceles triangle. In the 7th grade of our school, the equality of angles is determined by the operation "superposition", and student denotes the angles on the fragment simply by B and C. Therefore, after the triangles are equal on three sides (which is not specified in the fragment), the student immediately concludes: angle B is equal to angle C.

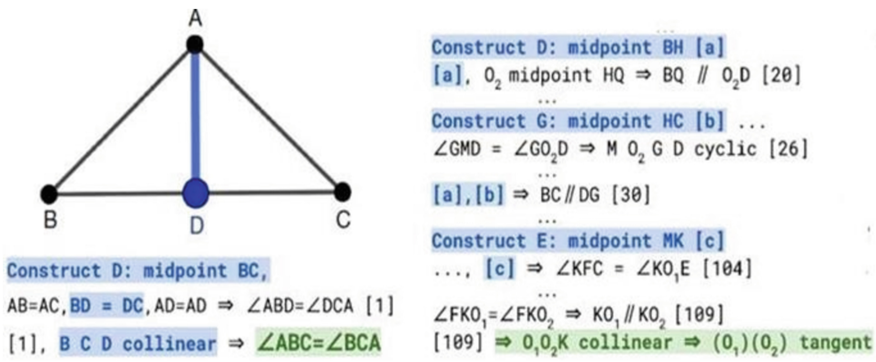

Fig. 1. Fragments to explain AlphaGeometry operation.

The right part is much more complicated, it describes AlphaGeometry's actions when solving an Olympiad problem very concisely: "Let ABC be an acute triangle. Let (O) be its circumcircle, H its orthocenter, and F the foot of the altitude from A. Let M be the midpoint of BC. Let Q be the point on (O) such that QH ⊥ QA and let K be the point on (O) such that KH ⊥ KQ. Prove that the circumcircles (O1) and (O2) of triangles FKM and KQH are tangent to each other." [18].

The drawing formed according to the task conditions is generated in the system with the help of visualization functions described above. The drawing is very cumbersome and causes cognitive discomfort, which is overcome in the system with the help of "hide/show" functions. These functions ensure that only key elements of the drawing are visible, such as those required to apply some theorem. In the process of experiments, the solution of several hundreds of tasks selected in the aspect of overcoming CD (difficulties in understanding the text, multiple solutions, leading to absurdity - proof from the contrary, false conclusions from the drawing, etc.) was investigated.

The choice of the Olympiad task (AlphaGeometry) is conditioned both by the fact that it is an important achievement in the field of automatic problem solving and by the complex of cognitive difficulties in solving it by a human. We emphasize that the system did not obtain a complete solution of the problem, but partial results turned out to be very interesting. In particular, the system found empirically and used to call the cognitive scheme "angles with mutually perpendicular sides", which was further used in the proof. AlphaGeometry's statement about the equality of other angles was evaluated as incorrect.

We emphasize that the experiments involve computer modeling of the CD. At the linguistic level, the text of the task (in Russian) reduces the EL description to a standard form by paraphrasing [12] (simulation of "understanding"). In this case, overcoming CD, in particular, relies on the knowledge of definitions ("the segment from the vertex to the middle of the side is the median", similarly for bisectors and heights). Plausible reasoning (Poya-style) allows one to overcome the CD when solving. For example, if angles 20° and 80° are given in a triangle, their difference (60°) is described in the ontology as more "familiar". This is an angle in an equilateral triangle, and it is "plausible" that it could be useful in the solution. The ontology provides dozens of schemas that formalize such plausible reasoning.

From Poya's point of view, the AlphaGeometry solution does not completely eliminate the problem of CD, even if every step is correct (there are more than 200 steps in the AlphaGeometry solution). For Poya, it is important to structure the solution, to organize it into observable, intuitively clear fragments connected by the simplest possible logic. Only in this case, according to Poya, we can talk about a full-fledged "understanding" of the proof. Our research is aimed at a computerized realization of this very concept of Poya.

At the level of visualization, overcoming CD is associated with improving the quality of the drawing (adapting it to the task conditions) and overcoming the discomfort of a cluttered drawing. In complex situations, overcoming CD is supposed to be done both by the student himself (in dialog with the system) and by the instructor.

4 Discussion

4.1 Testing

In general, the test results confirmed both the significance of the means of overcoming cognitive discomfort when solving geometric problems and revealed the direction of further improvement of such means.

The Olympiad problem considered above was discussed at seminars with schoolchildren. In the process of communication, they demonstrated NL-descriptions for drawing, JavaScript source code for visualization and possibilities of drawing modification with preserving the problem conditions. Most of the students with interest formed the drawing, supplementing and modifying the result of visualization. Two students, after demonstration of the drawing of the Olympiad problem and possibilities to modify it without violating the restrictions, asked a question that required fundamental discussion. The question was, "Why do we need a proof? After all, the experiment confirms with great accuracy what we want to prove".

The discussion emphasized the limitations of experimentation (both on a physical level - instrument, precision, etc., and on a philosophical level - divisibility of matter). The discussion of limitations revealed to the students the qualitative difference between experimental data and abstractions based on them, the role of such a difference in non-Euclidean geometry. In particular, while the formulation about the uniqueness of a parallel line through a point outside a straight line was considered by the students as obvious, the equivalent formulation - the sum of angles of a triangle equals 180° - was considered by the students rather as requiring experimental verification.

The ontological justification of the relationship between axiomatics and experimental data is not only true in geometry, it is even more important in physics (the limits of divisibility and density of matter, the maximum speed of a material object and its consequences for the concepts of "time" and "mass"). Therefore, a student who knows the physical limit on the maximum speed of a material body (the speed of light) can more easily overcome the CD arising from such a consequence of Lobachevsky's geometry as the limit on the maximum area of a triangle.

4.2 Related Works

The rapid trend towards the use of neural networks and Large Language Models (LLM) is of particular concern in social domains (healthcare, education, law enforcement, etc.). Psychological aspects of the cognitive dissonance associated with LLM and the dangers of using AI in social domains are discussed in [5]. The study [8] rightly points out that although big data and deep learning technologies dominate the IT industry, a source such as expert knowledge has not lost its relevance. Intensification of research and development in knowledge engineering will be able to synergize with the integration of knowledge from different sources [8].

Let us note a number of related works, more related to the solvers themselves. The Open Geometry project [1] proposes to integrate the efforts of various researchers on the problem of automating proofs. In [6] the importance of geometric reasoning with computer applications for future knowledge accumulation and dissemination is emphasized. NL-analysis and synthesis based on large language models, focusing on Boolean logic and first-order predicate languages is discussed in [7]. In [15], a Geometry3K benchmark consisting of 3002 geometric problems densely annotated in a formal language is described. In [17], an analysis of elementary school math Olympiad problems is given as a possible benchmark of language and math comprehension tests.

In our system, the corpus of solved problems is assumed to be stored in an ontology. The corpus includes a formal representation of the problem with the solution, a protocol with NL-descriptions of the steps, and interactive visualization tools. The problem of overestimating-underestimating deductive reasoning in the aspect of a tutoring program with AI is discussed in [3]. In [4], the problems and selected solutions for generating a proof in Prolog language are discussed, but oriented towards educational goals. In [14], a neural network training algorithm for theorem proving search (HyperTree Proof Search) was proposed.

Note that CD and its overcoming in clearly formalized domains (geometry), will, in our opinion, enable the development of thinking skills to analyze a much wider range of issues. A critical view of phenomena not at all related to mathematics, a clear understanding of the differences in the basic principles (axioms), will be necessary for the future specialist regardless of his professional activity. In general, it is necessary to note the great potential of the system under study to promote elements of artificial intelligence directly into school education.

5 Conclusion

The scientific significance of the work consists in the possibility to take a new look at the cognitive mechanisms and models of eliminating cognitive dissonance, having generalized the specific results of solving planimetric problems within the framework of a computer system. The necessity of an integrated approach to overcoming cognitive dissonance at all stages of interaction with the system - linguistic, logical, visual - is noted.

In the applied aspect, the introduction of the research results into practice will make it possible to introduce students to the elements of artificial intelligence in very specific situations directly related to the educational process ("understanding" by the computer of the task text, automation of the solution and its visualization) already at the early AQ2 stages of education. It is equally important that students can directly participate in the modification and development of the system, since the source code of the programs is open.

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Chapter 15

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